

# 3D GRAPHING QUADRATIC EQUATIONS WITH SIMULATION METHODOLOGY

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## Abstract

This paper presents a method of simulating 3D quadratic surfaces produced by solving second degree equations in three variables, and projecting the graph on a two dimensional display. The method adopts a simple coordinate system that can be redefined to produce different 3D shapes that reflect the art of the mathematical equations.

**Keywords:** Graphic, Equation.

## Introduction

Sketching a portrait by an artist could be effected by many factors such as; the artist background, the social and psychological state, the subjective idea beside his own style. but, we can see astonishing portrait that can amaze any artist drawn with or without intention, and does not effected by those factors. These drawings like the ones that are presented in this paper lies within this category and are constructed according to a scientific bases instead of the artist vision. The scientific idea of these shapes could not be conceived by the ordinary viewer unless he has a scientific or mathematical depth, but in any way the beauty of these shapes can be visualized easily. Therefore we may call this type of art as a scientific art.

## Quadratic Surfaces

The graphs of second degree equation in two variables such as;  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$  is a conic section that is a parabola, ellipse, circle or hyperbola. The graph of a second degree equation in three variables;  $Ax^2 + By^2 + Cz^2 + Dx + Exz + Fyz + Gx + Hy + Iz + J = 0$ , that can be reduced by rotation to be,  $Ax^2 + By^2 + Iz^2 = 0$  is called a quadratic surface. There are nine distinct quadratic surfaces that are identified by using a standard equation. For those types that have centers, the center will be at the origin; for those that are symmetric, the axis of symmetry will be a coordinate axis, as seen in the coming figures.

The quadratic surface is characterized by citing its intercepts (points, at which the coordinate axes are crossed); its traces (the intersection of the surface with the coordinate planes), and its sections (the intersection of the surface with the planes, usually taken parallel to the coordinate planes).

The equation:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  represent an ellipsoid. The intercepts are the points  $(\pm a, 0, 0)$ ,  $(0, \pm b, 0)$  and  $(0, 0, \pm c)$ , the traces are all ellipses. For example, the trace in x-y plane is the ellipse  $(\frac{x^2}{a^2} + \frac{y^2}{b^2}) = 1$ . All sections parallel to the coordinate planes are ellipses if  $a \neq b$ , and are circles if  $a = b$ , were  $z = 0$ .

### 3D Projection

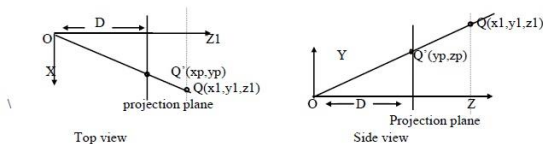
Usually in graphics, a three-dimensional object is viewed on a two-dimensional display screen. A projection is a transformation operation that performs this conversion from three-dimensional representation to a two-dimensional representation. This process takes different techniques depending on the field of computer graphics. Artists use a perspective projection to create a realistic pictures; architects use several parallel projection to obtain views of a building; cartographers use macerator and conical projections to draw maps, and there are many other types and their users. Our concern here is the perspective projection.

### Perspective Projection

This projection reflects the real-world, where object closer to an observer look larger than those in the background, and this concise with what human eye and photographic lenses can see. This technique conveys depth information by making distance objects smaller than near ones, and the eye is connected with the object by unparallel projectors cross the projection plane and converge at the eye.

Let us assume that the projection plane is parallel to x-y plane at a distance D from the eye. The eye represents the center of projection.

Q is projected to Q' at the projection plane.



By proportion property of similar triangles we obtain:

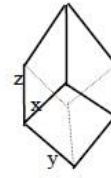
$$\begin{aligned} X_p/D &= X_1/Z_1 & \text{or} & \quad X_p = D * Y_1 / Z_1 \\ Y_p/D &= Y_1/Z_1 & \text{or} & \quad Y_p = D * Y_1 / Z_1 \end{aligned}$$

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The projection technique that adopted in this paper is the following: By drawing the cube in Fig-3, including hidden edges with dotted lines, selecting a corner of the cube as the origin and label the three lines emanating from that point x, y, and z. By this we have a system for projecting three dimensional points on to two dimensions. For example, to locate the point (6,8,3), start at the origin and move 6 units along the x-axis from that point, move 8 units in a direction parallel to the y-axis and then move 3 units in a direction parallel to z-axis Fig(4). The

orientation of the axes can be varied by rotating the projected objects.



3D coordinate sys.  
in 2D plane.  
Fig-3.



point (6,8,3) in  
the sys.  
Fig-4.

To project three-dimensional object on the display screen, we describe the origin and the axis in terms of the display's horizontal and vertical coordinate system (H and V). Each of the three axis is defined by the angle it forms with a horizontal line extending from the origin in a positive direction. ( $\theta_x, \theta_y, \theta_z$ ). Using these basics with elementary trigonometry, its possible to project any point (x, y, z) on the display in terms of H and V.

For point (x,0,0),  $H=x \cos(\theta_x)$ ,  $V=x \sin(\theta_x)$

Similarly for point (0,y,0),  $H=y \cos(\theta_y)$ ,  $V=y \sin(\theta_y)$

for point (0,0,z),  $H=z \cos(\theta_z)$ ,  $V=z \sin(\theta_z)$

For point(x, y, z), the horizontal and vertical coordinate are summed:

$$\begin{aligned} H &= x \cos(\theta_x) + y \cos(\theta_y) + z \cos(\theta_z) \\ V &= x \sin(\theta_x) + y \sin(\theta_y) + z \sin(\theta_z) \end{aligned}$$

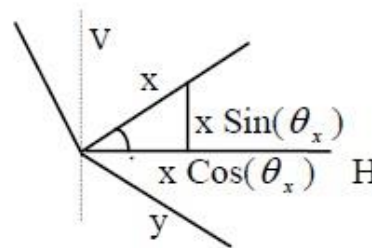


Fig-5.

### Plotting Method

Functions in three variables are convenient source of data points that form a fascinated and unlimited variety of shapes and patterns. In fact there aren't any shape with planned features can be generated,

but the stubbornly abstract that are formed may present a fine arts. Generally the usual method used is to consider  $z$  as a function of  $x$ , and  $y$ . For instance the equation;  $x^2 + y^2 + z^2 = 4$  is rearranged to get  $z$  as:

$$z = \pm\sqrt{4 - x^2 - y^2}$$

To plot the function, we scan the  $x$ - $y$  plane at regular intervals, selecting values for  $x$  and  $y$ , and calculate resulting value for  $z$ . To remove the hidden surfaces, complicated hidden-line and hidden-surface algorithms may be used, but in this respect we achieve this by drawing the background before the foreground, so that it obscures background points. The presented program also calculates the foreground points closer together and assigns different colors to foreground and background surfaces. All these techniques improve the apparent three-dimensionality of the rendering.

Figures 1 through 6 are the graphs of common second-degree equations in three variables. The images of these graphs are produced on a PC computer with screen resolution of 512 by 512 pixels.

#### Program Notes

The main program (listing-1) begin by asking to specify the orientation of  $x$ ,  $y$  and  $z$  axes in degree. Positive or negative values may be entered, so that positive angles are rotated counterclockwise and negative angles are rotated clockwise around the horizontal axis. The figure are created using angles of 15, -15, and 90 for  $x$ ,  $y$ , and  $z$  respectively. To render a surface in the other orientation, we simply change one or more of the axes.

Line 7 define the function which includes two arguments,  $x$ , and  $y$ ; and returns a value to be used for  $z$ .

Line 57-58 specify the origin in term of screen coordinates  $HC$ , and  $LC$ . The position is calculated as fraction of the number of points in each dimension.  $HC$  is halfway across the screen ( $HRES/2$ ) and  $(19/20)$  of the way down the screen ( $19*LRES/20$ ).

Line 59-60 set the horizontal and vertical scaling factors, which a surface that it can fits within the plotting area.

Lines 81-92 make up the plotting routine. The program steps through a domain of  $x$  and  $y$  values, calculates a  $z$  for each  $x$  and  $y$ , and plot the point. Colors are also changed for points for which  $x$  is negative; this help in the simulation of three-dimensionality.

Graphing a different function will require minor changes to the main program. Listing 2 through 6 consist of all changes that will be needed to make general figures 2 through 6. It is interesting to explore generalization more intricate figures by applying an extra transformation to one or more of the coordinates before each point is plotted. As in figure 7 and figure 8.

#### References

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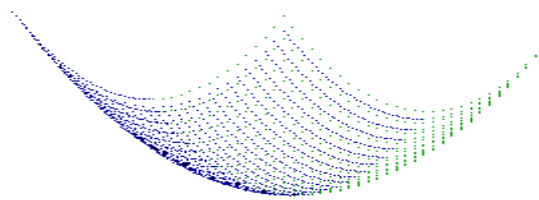


Figure 1: Paraboloid  $x^2/A^2 + y^2/B^2 - z^2/C^2 = 0$

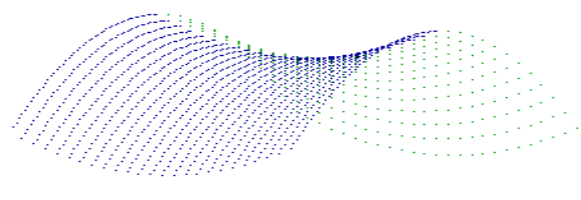


Figure 2: Hyperboloid paraboloid,  $y^2/A^2 - x^2/B^2 - z^2/C^2 = 0$

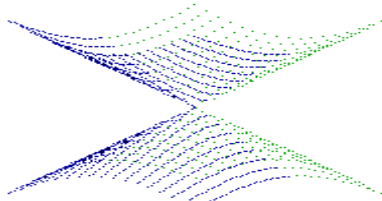


Figure 3: Cone,  
 $x^2/A^2 + y^2/B^2 - z^2/C^2 = 0$

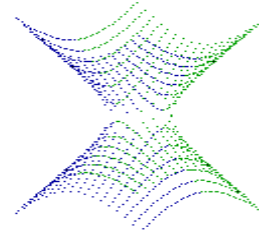


Figure 4: hyperboloid of one sheet,  
 $x^2/A^2 + y^2/B^2 - z^2/C^2 = 1$

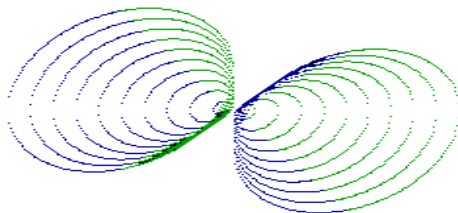


Figure 5: Hyperboloid of two sheets,  $x^2/A^2 - y^2/B^2 - z^2/C^2 = 1$

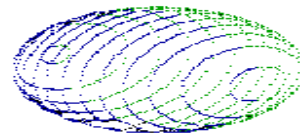


Figure 6: Ellipsoid,  $x^2/A^2 + y^2/B^2 + z^2/C^2 = 1$

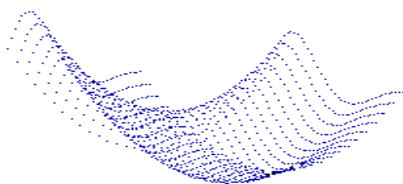


Figure 7: Wavy Paraboloid.

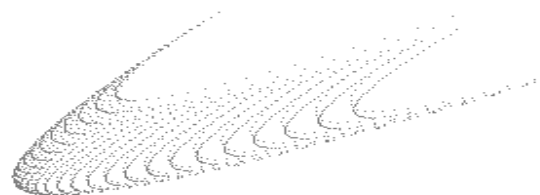


Figure 8: Comet